Indian J. Phys. 44, 392–398 (1970)

Čherenkov type of radiation in an anisotropic electron plasma

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(*Received* 26 *February* 1970—*Revised* 26 *May and* 31 *July* 1970)

An exact solution for the electromagnetic field due to a moving line charge through an anisotropic electron plasma in the presence of an external magnetic field is given. Electromagnetic intensities and radiation are slowly damped due to collision of electrons but the attenuation is very intensive within a certain range of frequency. The wave propagation is mainly transverse to the dc magnetic field and there exists other type of radiation along with the Čherenkov radiation

Introduction

Kolomenskii (1956) investigated the Cherenkov ratiation in an anisotropic electron plasma under an external magnetic field without collision effect. Majumdar (1961) studied a similar problem in a homogeneous electron plasma. He investigated the possibility of other types of radiation in addition to the Čherenkov radiation due to coupling between the longitudinal plasma waves and transverse electro-magnetic waves. Some aspects of the field in a collisionless plasma due to a line source have been considered by Wait (1960), Tuan & Seshadri (1965) and many others.

As a sequel to these works we shall study the interaction of an uniformly moving line charge with the transverse electromagnetic waves in an infinite magneto-electron plasma, which is incompressible but anisotropic in dielectric property, with the low collision frequency. In our case the source is a line charge moving perpendicularly to the external magnetic field. Due to this ideal consideration the problem becomes two-dimensional and the longitudinal wave is trivial. The uniform velocity of the source introduces a cut-off in the frequency range for propagation of waves and the collisions of electrons include damping of the electromagnetic intensities and radiation. It is found that the radiation is confined between two planes, somewhat similar to Čherenkov cone, and it exists even when the velocity of the source is less than the phase-velocity of light. As a limiting case the expression for radiation in the direction of Čherenkov ray matches with that of Čherenkov radiation due to the motion of a line charge within a homogeneous dielectric medium given by the author (1967). Similar phenomenon may occur in the ionosphere due to some sudden solar disturbances. Moreover, the moving source may be utilized like a test particle to explore the characteristics of the earth's ionosphere where the charged particles are moving at high speed across the magnetic field.

Phenomenological Equations

We assume that (i) ions are stationary and they neutralize the electrons on the average, (ii) an external magnetic field B_0 is acting in the direction of z-axis, (iii) a line charge is moving with uniform velocity u in the direction of x-axis, and (iv) the collision factor is a small quantity of the first order.

Maxwell's equations for field variables $\vec{E}$ and $\vec{H}$ (electric and magnetic vectors) with Fourier transform are

$$\nabla \times \vec{E}(\omega) = -\frac{i\mu\omega}{c} H(\omega) \quad \ldots \quad (1)$$

and

$$\nabla \times \vec{H}(\omega) = \frac{i\omega}{c} \vec{D}(\omega) + \frac{4\pi}{c} \vec{j}(\omega). \quad \ldots \quad (2)$$

μ c and $\vec{j}$ are the scalar magnetic permeability, the velocity of light in free space and the current density. $\vec{D} = (\chi)\vec{E}$ where (χ) is the dielectric tensor described

$$(\chi) = \begin{pmatrix} c_1 & -\epsilon' & 0 \\ \epsilon' & c_1 & 0 \\ 0 & 0 & c_2 \end{pmatrix}$$

Here

$$\left.\begin{aligned} \frac{c_1}{c} &= 1 - \frac{\omega_p^2}{\omega^2 - \omega_c^2} - i\eta\omega_p^2 \frac{\omega^2 + \omega_c^2}{\omega(\omega^2 - \omega_c^2)^2}, \\ \frac{c_2}{\epsilon} &= 1 - \frac{\omega_p^2}{\omega^2} - i\eta \frac{\omega_p^2}{\omega^3}, \quad \frac{\epsilon'}{\epsilon} = -\frac{i\omega_c\omega_p^2}{\omega(\omega^2 - \omega_c^2)} + \eta \frac{2\omega_c\omega_p^2}{(\omega^2 - \omega_c^2)^2}, \\ \omega_p^2 &= \frac{4\pi N e^2}{\epsilon m}, \quad \omega = \frac{eB_0}{mc} \end{aligned}\right\} \quad \ldots \quad (3)$$

and ϵ is the permittivity of the medium in the limit. The tensor (χ) is obtained by the help of the force equation,

$$m \frac{\partial \vec{V}}{\partial t} + m\eta\vec{V} = e\left[\vec{E} + \frac{\vec{V} \times \vec{B}_0}{c}\right]. \quad \ldots \quad (4)$$

N, e, m, η and $\vec{V}$ are the average electron number density, electron charge, mass of an electron, the collision factor and the velocity of the electrons, In case of $\vec{j}$, $j_y = 0 = j_z$ and $j_x = qu\delta(x-ut)\delta(y)$ where q is the line charge density By Fourier transform

$$j_x(x, y, \omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} qu\delta(x-ut)\delta(y)e^{-i\omega t}dt = -\frac{q}{2\pi}e^{-\frac{i\omega x}{u}}\delta(y). \tag{5}$$

The problem thus reduces to a two-dimensional one and all the vector quantities are independent of z, *i.e.*, the wave propagation is transverse to the applied magnetic field. Moreover, without any inconsistency in the equations (1), (2) and (4) we can assume that $E_z = 0 = H_x = H_y = V_z$. It means in effect absence of longitudinal plasma waves.

With the aid of (2) and (3) we have

$$\left.\begin{aligned} E_x(\omega) &= -\frac{ic}{c(\epsilon_1{}^2+\epsilon'^2)\omega}\left[\epsilon_1\frac{\partial H_z(\omega)}{\partial y} - \epsilon'\frac{\partial H_z(\omega)}{\partial x} - \frac{4\pi}{c}\epsilon_1 j_x(\omega)\right] \\ \text{and}\quad E_y(\omega) &= \frac{ic}{c(\epsilon_1{}^2+\epsilon'^2)\omega}\left[\epsilon'\frac{\partial H_z(\omega)}{\partial y} + \epsilon_1\frac{\partial H_z(\omega)}{\partial x} - \frac{4\pi}{c}\epsilon' j_x(\omega)\right]\end{aligned}\right\} \quad \dots (6)$$

By (1) and (6) we obtain

$$\frac{\partial^2 H_z(\omega)}{\partial x^2} + \frac{\partial^2 H_z(\omega)}{\partial y^2} + \frac{\mu\epsilon}{c^2}\frac{\epsilon_1{}^2+\epsilon'^2}{\epsilon_1}\omega^2 H_z(\omega) = \frac{4\pi}{c}\left\{\frac{\epsilon'}{\epsilon_1}\frac{\partial j_x(\omega)}{\partial x} + \frac{\partial j_x(\omega)}{\partial y}\right\} \quad \dots (7)$$

Formal Solution of the Equation (7)

Considering Fourier transform with respect to y in the form $f(x, y, \omega) = \int_{-\infty}^{\infty} f(x, k, \omega)e^{iky}dk$ and in view of dependence of field components on x through the phase factor $e^{-\frac{i\omega x}{u}}$ the equation (7) gives

$$H_z(x, k, \omega) = \frac{iq}{\pi c}\,\frac{\frac{\epsilon'}{\epsilon_1}\frac{\omega}{u} - k}{\frac{\mu\epsilon}{c^2}\frac{\epsilon_1{}^2+\epsilon'^2}{\epsilon_1}\omega^2 - \frac{\omega^2}{u^2} - k^2}\, e^{-\frac{i\omega x}{u}}$$

$$H_z(x, y, \omega) = \frac{iq}{\pi c}\int_{-\infty}^{\infty}\frac{k - \frac{\epsilon'}{\epsilon_1}\frac{\omega}{u}}{k^2 - \left(\frac{\mu\epsilon}{c^2}\frac{\epsilon_1{}^2+\epsilon'^2}{\epsilon_1}\omega^2 - \frac{\omega^2}{u^2}\right)}\, e^{iky - i\omega x/u}\,dk \quad \dots (8)$$

Under the assumption about η and for a particular value of ω the equation (8) reduces to

$$H_z(x, y, \omega) = \frac{iq}{\pi c} \int_{-\infty}^{\infty} \frac{k-(\eta b_1 - ia_1)}{k^2-(a^2-i\eta b)} e^{iky-i\omega x/u} dk \qquad \dots \quad (9)$$

where

$$a_1 = \frac{\omega_c \omega_p^2}{u h_2}, \; b_1 = \frac{a_1}{h_2} \frac{2\omega^2-\omega_p^2}{\omega}, \; a^2 = \frac{\mu\epsilon}{c^2}\frac{h_3}{h_2} - \frac{\omega^2}{u_2},$$

$$b = \frac{\mu\epsilon}{c^2} \frac{\omega\omega_p^2}{h_1}, \; h_1 = \omega^2-\omega_c^2, \quad h_2 = \omega^2-\omega_c^2-\omega_p^2, \; h_3 = (\omega^2-\omega_p^2)^2-\omega^2\omega_c^2 \qquad \dots \quad (10)$$

On taking $a^2 > 0$ and integrating (9) by the residue method

$$\left.\begin{aligned} H_z(x, y, \omega) &= -\frac{q}{c}\left[1-\frac{ia_1}{a}+\eta\xi\right] e^{\psi_1} \text{ when } y > 0 \\ \text{and} \quad H_z(x, y, \omega) &= -\frac{q}{c}\left[1+\frac{ia_1}{a}-\eta\xi\right] e^{\psi_2} \text{ when } y < 0. \end{aligned}\right\} \qquad \dots \quad (11)$$

$$\left.\begin{aligned} \text{Here} \quad \xi &= \frac{b_1}{a}+\frac{ba_1}{2a^3}, \; \psi_1 = -(\alpha+i\gamma)y-\frac{i\omega x}{u}, \\ \psi_2 &= (\alpha+i\gamma)y-\frac{i\omega x}{u}, \; \sqrt{a^2-i\eta b} = \gamma-i\alpha, \; \gamma \simeq a, \; \alpha \simeq \frac{\eta b}{2a}. \end{aligned}\right\} \qquad \dots \quad (12)$$

Expressions for $\dot{E}$ and $\dot{V}$

With the aid of (6) and (11) the following values of E_x and E_y are obtained.

$$\left.\begin{aligned} \text{when} \quad y > 0, \quad E_x(x, y, \omega) &= -\frac{iq}{\omega\epsilon a_2}[im_1+\eta(f_1+ig_1)]\, e^{\psi_1} \\ \text{and} \quad E_y(x, y, \omega) &= \frac{iq}{\omega\epsilon a_2}[l_2+im_2+\eta(f_2+ig_2)]\, e^{\psi_1}. \end{aligned}\right\} \qquad \dots \quad (13)$$

$$\left.\begin{aligned} \text{When} \quad y < 0, \quad E_x(x, y, \omega) &= \frac{iq}{\omega\epsilon a_2}[im_1+\eta(f_1-ig_1)]e^{\psi_2} \\ \text{and} \quad E_y(x, y, \omega) &= -\frac{iq}{\omega\epsilon a_2}[l_2-im_2+\eta(-f_2+ig_2)]e^{\psi_2} \end{aligned}\right\} \qquad \dots \quad (14)$$

Here

$$
\left.
\begin{aligned}
m_1 &= \frac{h_2}{ah_1}(a^2+a_1^2),\quad l_2 = \frac{\mu\epsilon}{c^2}\frac{ua_1}{\omega a}\frac{h_3}{h_1},\\
m_2 &= \frac{u}{\omega}\frac{h_2}{h_1}\left(\frac{w^2}{u^2}-a_1^2\right),\quad a_2 = \frac{h_3}{\omega^2 h_1^2},\\
f_1 &= h_4-\frac{a_1}{a}h_5-\frac{\omega_c\omega_p^2}{\omega h}\xi,\quad g_1 = \frac{ah_2}{h}\xi-\frac{a_1}{a}h_4-h_5,\\
f_2 &= \frac{\omega_p^2(\omega^2+\omega_c^2)}{uh_1^2}+\frac{a_1}{a}h_6+\frac{\omega_c\omega_p^2 a}{\omega h}\xi-\frac{2h_2\omega^2h_7}{u},\\
g_2 &= h_6+\frac{\omega}{u}\frac{h_2}{h_1}\xi+\frac{2\omega_c\omega_p^2}{au^2}h_7.\\
h_4 &= \frac{b}{2a}\frac{h_2}{h_1}+\frac{a\omega_p^2(\omega^2+\omega_c^2)}{\omega h_1^2}-2a\omega h_2h_7,\quad h_5 = \frac{2\omega\omega_c\omega_p^2}{u}\left(\frac{1}{h_1^2}+h_7\right),\\
h_6 &= \frac{\omega_c\omega_p^2}{h_1}\left(\frac{2a}{h_1}-\frac{b}{2a\omega}\right)+2a\omega_c\omega_p^2h_7,\quad h_7 = \frac{\omega_p^2(\omega^2+\omega_c^2-\omega_p^2)}{h_1^2h_3}.
\end{aligned}
\right\} \quad \dots (15)
$$

THE NATURE OF THE RADIATION

The power radiated by the moving source per unit time is

$$
\vec{S} = \frac{c}{4\pi}.2Re\int_{-\infty}^{\infty}(\vec{E}\times\vec{H})dx
$$

$$
= cu\ Re\int^{\infty}(\vec{E}_\omega\times\vec{H}_\omega)d\omega.
$$

When $y > 0$, $S_x(\omega) = \frac{q^2u}{\omega\epsilon a_2}\left[\frac{a_1l_2}{a}+m_2+\eta\left(m_2\xi+\frac{a_1f_2}{a}+g_2\right)\right]e^{-2\alpha y}$ (16)

and $S_y(\omega) = q^2u\left[m_1+\eta\left(m_1\xi+\frac{a_1f_1}{a}+g_1\right)\right]e^{-2\alpha y}$ (17)

with the help of (11) and (13).

When $y < 0$, $S_x(\omega) = \frac{q^2u}{\omega\epsilon a_2}\left[\frac{a_1l_2}{a}+m_2-\eta\left(m_2\xi+\frac{a_1f_2}{a}+g_2\right)\right]e^{2\alpha y}$ (18)

and $S_y(\omega) = \frac{q^2u}{\omega\epsilon a_2}\left[m_1-\eta\left(m_1\xi+\frac{a_1f_1}{a}+g_1\right)\right]e^{2\alpha y}$ (19)

with the help of (11) and (14). Here $S_y(\omega)$ is in the negative direction of y-axis.

It is of interest to examine the angular distribution of the radiated energy. The angles between the direction of motion of the line charge and the Čherenkov rays are given by

$$\tan\,\theta_1 = \frac{S_y(\omega)}{S_x(\omega)} = \frac{ua}{\omega}\,[1+\eta\xi_1] \qquad \ldots \quad (20)$$

and

$$\tan\theta_2 = \frac{ua}{\omega}\,[1-\eta\xi_1] \qquad (21)$$

according as $y>0$ or $y<0$.

Here

$$\xi_1 = \frac{1}{m_1}\left[\, m_1\xi + \frac{a_1 f_1}{a} + g_1 - \frac{ua}{\omega m_1}\left(m_2\xi + \frac{a_1 f_2}{a} + g_2\right)\right].$$

From (20) and (21) it is clear that the intensities of radiation in the two zones of y make different angles with the line of the moving source.

Conclusion

Th equations of electromagnetic intensities and the radiated energy reveal that they are exponentially damped with y. Since η is very small, the penetration depth $1/\alpha$ is fairly large and consequently attenuation takes place very slowly. Collisions of electrons are mainly responsible for damping when $a^2>0$. It is interesting to note that there will be no discrimination in the absolute values of field components, radiation and angular distribution for positive and negative zones of y unless collisions of electrons are taken into account.

From the equations of (11), (13) and (14) we see that outgoing plane waves will propagate so long as $a^2>0$. This Čherenkov like condition introduces a cut-off in frequency. In Čherenkov region, *i.e.*, where $\mu\epsilon\beta^2-1>0$, $(\beta = u/c)$, $a^2<0$ when $\omega_1<\omega<\omega_2$ and consequently there will be almost no wave propagation. The values of ω_1 and ω_2 are given by

$$(\omega_2^2,\, \omega_1^2) = \frac{\{\mu\epsilon\beta^2(2\omega_p^2+\omega_c^2)\omega_p^2+\omega_c^2\} \pm \sqrt{[\{\mu\epsilon\beta^2(2\omega_p^2+\omega_c^2)+\omega_p^2+\omega_c^2\}^2 - 4\mu\epsilon\beta^2\omega_p^2(\mu\epsilon\beta^2-1)]}}{2(\mu\epsilon\beta^2-1)}$$

respectively. In the non-Čherenkov region, *i.e.*, where $\mu\epsilon\beta^2-1<0$, the wave propagation as well as emission of radiation exist, if

$$\omega^2 > \frac{\sqrt{[\{\mu\epsilon\beta^2(2\omega_p^2+\omega_c^2)+\omega_p^2+\omega_c^2\}^2 + 4\mu\epsilon\beta^2\omega_p^2(1-\mu\epsilon\beta^2)]} - \{\mu\epsilon\beta^2(2\omega_p^2+\omega_c^2)+\omega_p^2+\omega_c^2\}}{2(1-\mu\epsilon\beta^2)}$$

These are not in accord with the usual Čherenkov phenomenon.

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When $y > 0$, $$S_x(\omega) = \frac{q^2u}{\omega\epsilon a_2}\left[\frac{a_1 l_2}{a}+m_2+\eta\left(m_2\xi+\frac{a_1 f_2}{a}+g_2\right)\right]e^{-2\alpha y} \quad (16)$$

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with the help of (11) and (14). Here $S_y(\omega)$ is in the negative direction of y-axis.

It is of interest to examine the angular distribution of the radiated energy. The angles between the direction of motion of the line charge and the Čherenkov rays are given by

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From the equations of (11), (13) and (14) we see that outgoing plane waves will propagate so long as $a^2 > 0$. This Čherenkov like condition introduces a cut-off in frequency. In Čherenkov region, *i.e.*, where $\mu\epsilon\beta^2-1 > 0$, $(\beta = u/c)$, $a^2 < 0$ when $\omega_1 < \omega < \omega_2$ and consequently there will be almost no wave propagation. The values of ω_1 and ω_2 are given by

$$(\omega_2^2,\ \omega_1^2) = \frac{\{\mu\epsilon\beta^2(2\omega_p^2+\omega_c^2)\omega_p^2+\omega_c^2\} \pm \sqrt{[\{\mu\epsilon\beta^2(2\omega_p^2+\omega_c^2)+\omega_p^2+\omega_c^2\}^2 - 4\mu\epsilon\beta^2\omega_p^2(\mu\epsilon\beta^2-1))]}}{2(\mu\epsilon\beta^2-1)}$$

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These are not in accord with the usual Čherenkov phenomenon.

It is evident from the expressions of electro-magnetic intensities that at a particular frequency radiation is more or less confined between two planes perpendicular to the wave planes parallel to $(\omega x/u)-\gamma y = 0$ and $(\omega x/u)+\gamma y = 0$. One may compare it with the Čherenkov cone. If 2θ be the angle between the normals to the above planes, then $\tan\theta \simeq ua/\omega$ which is identical with $\tan\theta_1$ or $\tan\theta_2$ for the zero value of η.

If we put $N = 0$ and $\eta = 0$ in (17), (19)' (20) and (21), then the interesting phenomena of Čherenkov radiation by a line charge in an infinite homogeneneous dielectric medium are obtained In this case $S_y = (\mu\epsilon\beta^2-1)q^2/\epsilon$ and $\tan\theta_1 = \tan\theta = \sqrt{\mu\epsilon\beta^2-1}$. These results have been worked out by the author (1967) for a non-magnetic medium (when $\mu = 1$).

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